Tidal current energy assessment for Johnstone Strait, Vancouver Island

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Abstract: The maximum tidal power potential of Johnstone Strait, BC, Canada is evaluated using a two-dimensional finite element model (TIDE2D) with turbines simulated in certain regions by increasing the drag. Initially, side channels are closed off so that the flow is forced through one channel to test the validity of a general analytic theory [**1**] with numerical results. In this case, the modelled power potential of 886 MW agrees reasonably well with the analytic estimate of 826 MW. In reality, two main channels, Discovery Passage and Cordero Channel, connect the Pacific Ocean to the Strait of Georgia. Turbines are simulated in Johnstone Strait, northwest of the two main channels, and separately for Discovery Passage and Cordero Channel. Northwestern Johnstone Strait is similar to the one channel case as the flow must go through this channel, but Discovery Passage and Cordero Channel are different as the flow can be diverted away from the channel with the turbines and into the other channel. The maximum extractable power in northwestern Johnstone Strait is found to be 1335 MW, which agrees well with the theoretical estimate of 1320 MW. In Discovery Passage and Cordero Channel, the maximum extractable power is modelled to be 401 and 277 MW, respectively, due to the flow being partly diverted into the other channel. In all cases, the current is reduced to between 57 and 58 per cent of the undisturbed flow, close to the 56 per cent predicted by the analytic theory. All power calculations are for the M2 constituent alone, as this is the largest current in the region. The total power from the eight major constituents (M2, S2, N2, K2, K1, 01, P1, and Q1) can be obtained by multiplying the power estimates for M2 by 1.12.

Keywords: tidal power, alternative energy, renewable energy, energy assessment

1 INTRODUCTION

Traditionally, tidal energy is extracted by blocking the entrance of a small bay with a barrier containing numerous sluices and turbines. The sluices allow the water to enter the bay on the flood tide but are closed at high tide. The water is then released through turbines when there is a large enough head difference due to the ebbing tide outside the barrage. A tidal power plant of this nature is located in the Bay of Fundy at Annapolis Royal with an installed capacity of 18 MW. A more complex scheme can generate electricity on both the flood and the ebb tide though this operates with a smaller head. The world's largest two-way generation tidal power plant is at La Rance, France, with an installed capacity of 240 MW and an average production of about 100 MW [**2**].

Although tidal power barrages produce zero emissions while operating, there are still many other environmental concerns [**3**], mostly due to a prolonged high tide inside the basin and a decrease in the tidal current speed. These concerns are reduced slightly with two-way generation, which more closely mimics the real tide, but there are still ecological impacts which will vary widely from site to site depending on the local tidal regime [**2**]. There may also be impacts on the tidal regime on the seaward side of the barrage [**4**], which could have serious implications for the neighbouring coastline.

In response to some of the environmental and ecological concerns with tidal barrages, there has

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*Corresponding author: *Department of Physics and Astronomy, P.O. Box 3055, STN CSC, Victoria, V8W 3P6, Canada. email: graig.sutherland@gmail.com* been a strong interest in harnessing power from tidal currents in a similar fashion to wind power generation [**5**]. This is seen as both a cheaper and more ecologically sound alternative to building a large tidal barrage. In 2002, Marine Current Turbines Ltd. (http://www.marineturbines.com) installed a single turbine with a rated capacity of 300 kW off Lynmouth, Devon, UK. This turbine only generates electricity for the tidal current moving one way, but there are proposals for a two-way rotor with a 1 MW rated capacity off the coast of Northern Ireland. Small tidal current projects are also underway at Race Rocks, BC, Canada (http://www.racerocks.com/racerock/energy/tidalenergy/tidalenergy.htm) and in the East River, New York City (http://www.verdantpower.com) with plans to install only one or two turbines. The majority of these projects are small scale (i.e. <1 MW) and will have little impact on the tidal currents at the site. However, for larger projects the turbines will tend to block the flow, thus reducing the power generated. Studies on how much energy can be extracted and the impacts on the tidal regime vary widely [**1**, **5–8**].

A common approach for evaluating tidal current potential is to assume that some percentage of the kinetic energy flux of the tidal flow (i.e. $(1/2)\rho u^3 \times$ the cross-sectional area) can be extracted for commercial use. An often quoted limit is the Betz limit [**9**], which claims that a maximum of 59 per cent of this kinetic power is available for extraction. However, this may be unrealistic because of the various assumptions made by Betz [**9**]. For example, Gorban *et. al.* [**10**] argued that only 35 per cent of the power may be extracted if one allows for the curvature of streamlines around the turbine. Regardless of what efficiency factor is used, the u^3 dependence (and a linear dependence on cross-sectional area) suggests that placing the turbines in the narrowest part of a confined stream will produce the most power. This may be true for an isolated turbine, or even a few turbines, as long as there is little change to the existing flow. However, it cannot apply to larger scale projects where there may be an appreciable change to the underlying flow because of the extra drag from the turbines. As more turbines are added, the flow is reduced, ultimately to the point at which the power produced decreases.

This article estimates the maximum power potential of the Johnstone Strait region (Fig. 1) in British Columbia, Canada. This region has very high tidal currents and has been proposed as an excellent site for harnessing tidal current energy [**5**]. First, theoretical calculations concerning the extraction of energy from the tidal currents are reviewed. The theory will then be applied to Johnstone Strait and compared with results from a numerical tidal model of the region.

2 A THEORETICAL APPROACH

For a single channel between two large basins, a general analytic theory to estimate the maximum extractable tidal power has been developed [**1**]. Turbines are simulated by increasing the friction across the entire cross-section of the channel. Assuming the wavelength of the tide is much longer than the channel length, so that the volume flux is constant along the entire channel, and the height difference between the ends of the channel does not vary with the addition of extra friction, the estimated maximum extractable power for a single tidal constituent was shown by Garrett and Cummins [**1**] to be given by

$$P_{\max} = \gamma \rho g a Q_{\max} \tag{1}$$

Here ρ is the density of seawater, g is the acceleration due to gravity, a is the amplitude of the sinusoidal height difference between the ends of the channel, and $Q_{\max}$ is the maximum volume flux in the natural tidal regime. All calculations assume the constant values for ρ and g of 1025 kg/m^3 and 9.81 m/s^2, respectively. The coefficient γ only varies over the small range between 0.20 and 0.24 and is determined by whether the forcing is balanced by acceleration or friction in the natural state without added drag from turbines.

The appropriate value of γ in a given situation may be determined by examining the phase lag of the current behind the maximum elevation difference in the natural state. If the forcing is balanced by acceleration, the phase lag is 90° and $\gamma = 0.24$, whereas if the forcing is balanced by friction and/or the effect of flow separation, and the response thus quasi-steady, the phase lag is zero and $\gamma = 0.21$. There is a small dip to a minimum $\gamma = 0.196$ for intermediate situations (see Fig. 4 of Garrett and Cummins [**1**]). For Johnstone Strait, the along-strait average phase lag is 35°, which is in between the two limits but closer to the quasi-steady case. A value of 0.20 is appropriate for γ and is used in this article.

Garrett and Cummins [**1**] showed how the quasi-steady limit can be examined analytically. In this case, the potential power P as a function of the peak flow rate, Q, with added turbines may be written

$$P/P_{\max} = \left(\frac{3^{3/2}}{2}\right)\left(\frac{Q}{Q_{\max}}\right)\left[1 - \left(\frac{Q}{Q_{\max}}\right)^2\right] \tag{2}$$

where $P_{\max}$ is the maximum power given by equation (1) and, as already defined, $Q_{\max}$ is the maximum volume flux in the natural tidal regime. The essential physics of the situation is

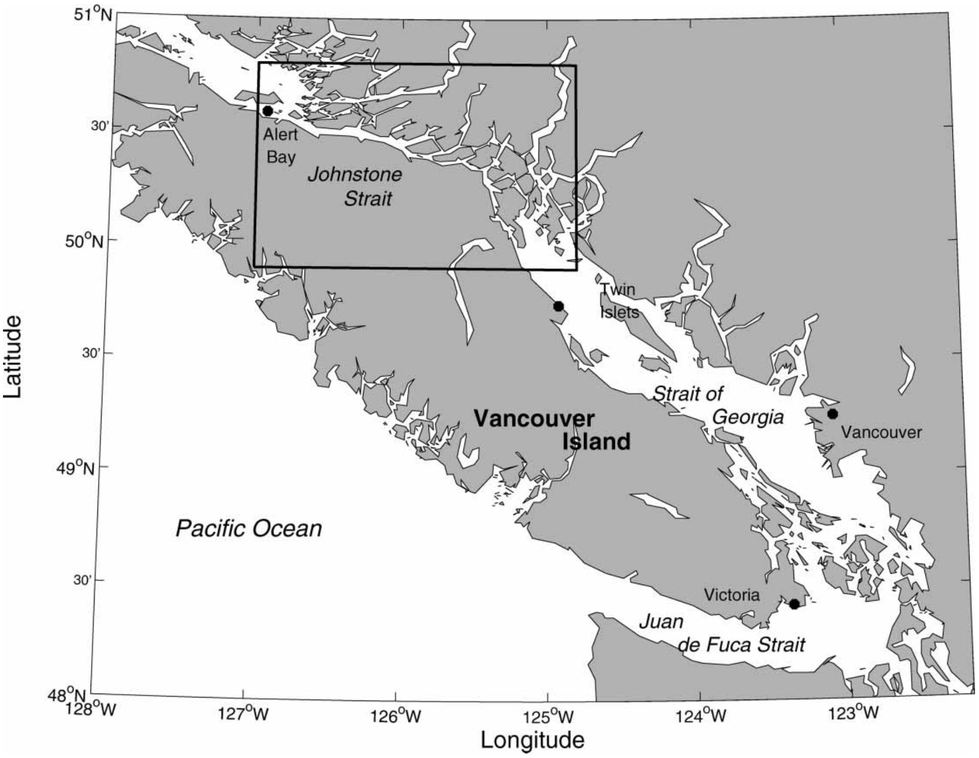


Fig. 1 Map of Vancouver Island. The location of the inset map (Fig. 3) is shown with the solid black line

well illustrated by equation (2). The term inside the square brackets is a non-dimensional form of the head loss across the fence of turbines, or an array of fences, and is zero when there are no turbines so that $Q = Q_{\max}$. As the number of turbines increases, this head loss increases as Q decreases. The power is given by the head times the volume flux. Ultimately, when so many turbines have been added that the flow is completely blocked, all the head loss originally associated with the natural state is transferred to the turbine array but the power produced is zero as there is no flow! Figure 2 is a graph of equation (2) illustrating this increase and then decrease of P as Q decreases from $Q_{\max}$ to zero. At maximum power extraction, the volume flux drops to 58 per cent of that in the natural regime and 2/3 of the original head along the whole channel has been transferred to the turbine array. For a 10 per cent reduction in the current, which might be more acceptable for environmental reasons than the 42 per cent decrease at the maximum, the available power is still 44 per cent of equation (1) with $\gamma = 0.21$. A very minor point worth mentioning is that the current referred to here is the current averaged over some suitable time. For low energy extraction, the reduction in this average current may be less than the magnitude of turbulent current fluctuations.

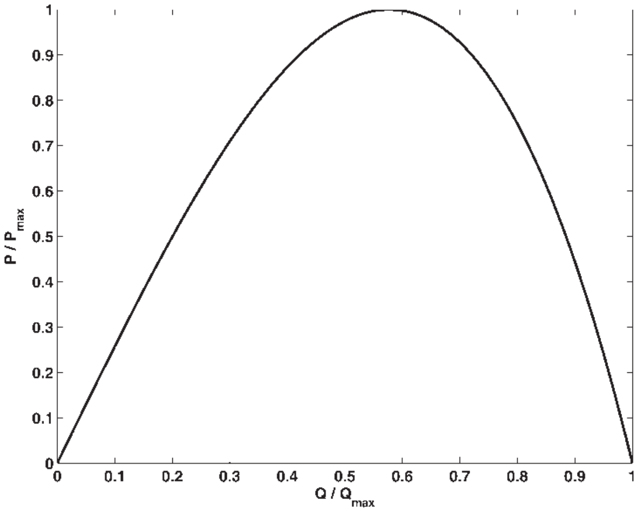


Fig. 2 The variation in the extractable power as a function of the reduced volume flux due to the presence of turbines for the situation in which there is a quasi-steady force balance in the natural state between pressure head and friction [1]. The volume flux is expressed as a fraction of the peak volume flux in the natural state and the power as a fraction of the maximum that can be extracted

These simple results are for the situation in which the natural state has a balance between forcing and friction. Numerical results (P. Cummins, 2006, personal communication) show that in the other limit with the basic balance between forcing and acceleration, the current at maximum power is 70 per cent of that in the natural state. For our intermediate situation, this fraction is 56 per cent, close to the 58 per cent for the quasi-steady limit.

3 JOHNSTONE STRAIT REGION

Much of Johnstone Strait (Figs 1 and 3) has a typical width of 4 km and mid-channel depths up to 400 m, while Seymour Narrows and Cordero Channel have minimum widths of 0.8 and 0.5 km, respectively, and mid-channel depths as little as 50 m. These constrictions and the near 180° phase difference between tidal elevations in the northern Strait of Georgia and Queen Charlotte Strait create some of the largest tidal currents in the world. Current speeds in Seymour Narrows can reach 7.7 m/s [**11**], with the along-channel M2 current amplitude being 4.7 m/s [**12**]. Gillard Passage and Arran Rapids, at the southern end of Cordero Channel, have maximum current speeds of 5.7 and 6.7 m/s, respectively [**11**]. Although there are also substantial estuarine flows in the region because of the runoff of several rivers, of which the Fraser is the largest [**13**, **14**], these flow values will not be included in this study. Likewise, important baroclinic features associated with the tides [**13**] will be neglected by assuming a homogeneous density for all model simulations and power potential calculations.

4 A NUMERICAL APPROACH

Tidal heights and currents are calculated with the TIDE2D finite element model, which solves the two-dimensional shallow water equations with conventional hydrostatic and Boussinesq assumptions. Particulars of the numerical scheme are described in detail by Walters [**15**]. The model application uses the same triangular grid, encircling Vancouver Island, that was employed in Foreman *et al.* [**12**]. The model grid was created using the software package TRIGRID [**16**] and digital coastline and bathymetric data obtained from the Canadian Hydrographic Service and the National Oceanic and Atmospheric Administration. Grid element sizes were chosen to preserve important coastline and bathymetric features such as the numerous narrow channels in the Johnstone Strait region. Triangle sides range from 12 km in the ocean west of Vancouver Island to 130 m in Seymour Narrows. In order to ensure that volume transports within channels are accurate, the depths assigned to each node represent an average of nearby soundings.

Although many tidal constituents can be investigated, only M2 was employed here to speed up computation and to more easily relate the work to the theoretical results of Garrett and Cummins [**1**]. Moreover, the tidal currents for the region are predominantly semi-diurnal, so the M2 constituent will be the major contributor of extractable current energy.

For the natural tidal regime, a bottom friction coefficient of 0.007 was assumed everywhere except for eastern Juan de Fuca Strait and Discovery Passage, where the values were assumed to be 0.02 and 0.013, respectively. All coefficients are larger than normal to compensate for the inability of the model to represent correctly all the dissipation mechanisms, such as unresolved flow separation, with a conventional coefficient of 0.003 [**12**], and to allow for M2 being the only modelled constituent when in reality, the other constituents make significant contributions to the non-linear bottom friction. The values of the friction coefficients were chosen to obtain a good agreement between the modelled and observed tidal elevations since, for tidal power calculations, an important aspect of the tidal height field is the sinusoidal height difference, between the ends of the channel, which is forcing the current. The height difference is calculated between Alert Bay and Twin Islets (Fig. 1) where the observed M2 elevations are obtained from a harmonic analysis [**17**] of the recorded time series [**11**] at each location. The modelled value of 2.11 m is nearly identical to the observed value of 2.12 m.

4.1 Energy dissipation rate

One way to compute the dissipation is to take the difference between the energy flux into and out of a section of the channel. However, this can lead to a large relative error when the difference is small compared to the incoming and outgoing energy fluxes. Also, it has been shown [**18**] that, since TIDE2D employs a wave-equation formulation, volume is not always conserved locally. This leads to a continuity equation residual that contributes to the energy budget and remains a matter of concern with the finite element approach. Nonetheless, the success of the model in reproducing the natural tidal regime accurately in a number of regions [**19**, **20**] suggests that it is still reliable for estimates of the current and hence for direct calculations of the energy dissipation rate.

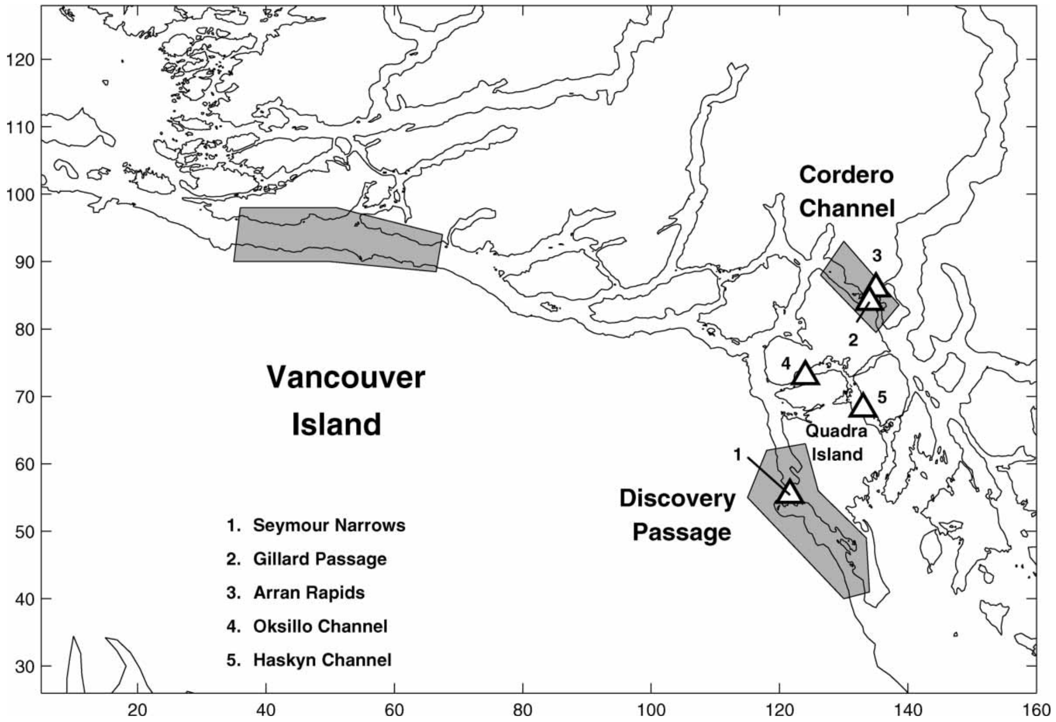


Fig. 3 Transect locations for the addition of synthetic turbines in Johnstone Strait. The shaded areas are the locations where turbines were simulated. The axes give the horizontal distances in kilometres

The rate at which energy is dissipated for a section of the seabed is calculated by integrating the bottom friction, that is,

$$P = \int\int_A \rho C_d \overline{|u^3|}\ \mathrm{d}A \tag{3}$$

where C_d is the quadratic bottom friction coefficient, u is the tidal current speed, and dA is an element of the area of the seabed.

5 TURBINE SIMULATIONS

The additional dissipation associated with the presence of turbines is simulated by increasing the bottom friction coefficient over a region of model nodes to represent a ‘farm’ of turbines. The bottom friction coefficient is increased to $C_d = k_0 + k_\mathrm{t}$ where k_0 is the natural bottom friction coefficient and k_t is that associated with the added turbines. The energy dissipated solely by the turbines is

$$P_\mathrm{t} = \frac{k_\mathrm{t}}{k_0 + k_\mathrm{t}} P \tag{4}$$

Figure 3 shows the areas of the simulated turbine farms. Using a two-dimensional numerical model it can be shown that these turbines extract energy from the entire cross-section of the tidal current flowing through a particular channel. This is similar to proposals for one or more tidal fences across the whole channel.

5.1 One Channel Open

To mimic the single channel case of Garrett and Cummins [**1**], we first close off the Cordero Channel and the Oksillo and Haskyn Channels on the eastern side of Quadra Island (Fig. 3) by disconnecting the nodes there. Although the system is still not exactly a single channel, because of some flow splitting in the central part of the strait, all the water entering into the Strait of Georgia must now pass through Discovery Passage and thus cannot be diverted away from the channel with the turbines. In this case, the height difference between Twin Islets and Alert Bay is 2.27 m.

Figure 4 shows the maximum M2 tidal volume flux for certain transects through Johnstone Strait before turbines are added. Along-channel variations in the volume flux are partly due to the limited accuracy of interpolating the volume flux to the transect location and partly due to the local accumulation or loss of water as the sea level changes with time.

The bottom friction coefficient is steadily increased in Discovery Passage until the extracted

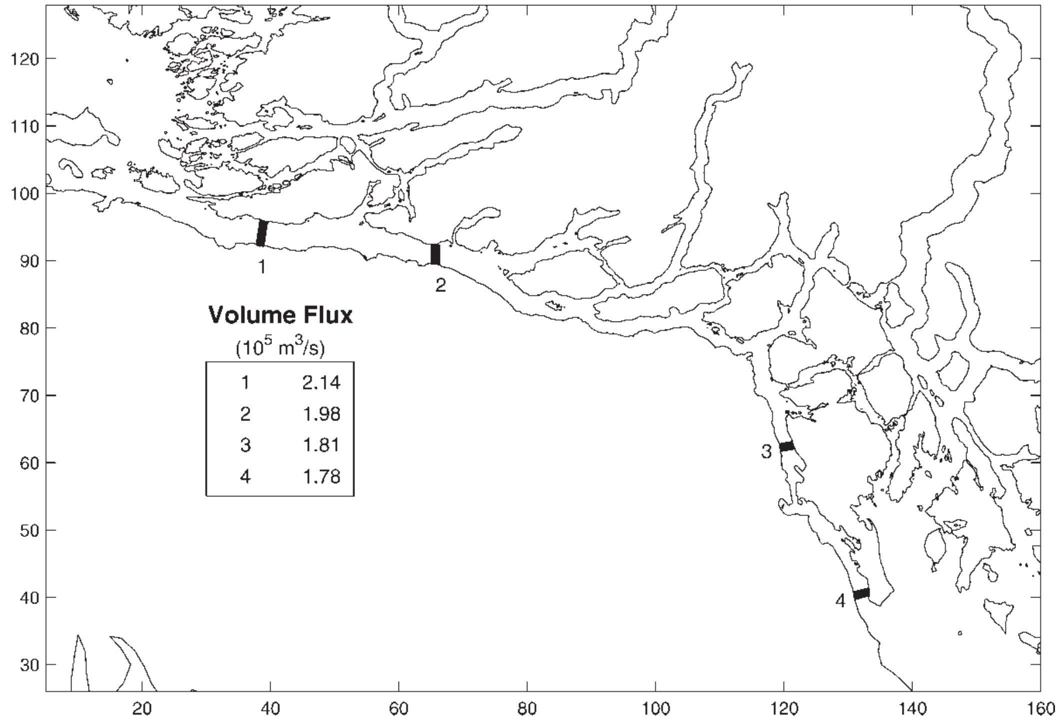


Fig. 4 Maximum volume flux with Cordero Channel effectively blocked and no turbines in Discovery Passage or Johnstone Strait. The axes give the horizontal distances in kilometres

energy, calculated using equation (4), peaks, as shown in Fig. 5. At peak power, the extracted energy is 886 MW with a corresponding drop in the maximum volume flux to 58 per cent of $Q_{\max}$, close to the theoretical expectation of 56 per cent cited earlier. The modelled power of 886 MW agrees reasonably well with the analytic value of 826 MW from equation (1) using $1.81 \times 10^5\,\mathrm{m^3/s}$ for the maximum volume flux, 2.27 m for a, and $\gamma = 0.20$. At peak power, a increases to 2.35 m, which is only a 3.5 per cent increase from the natural regime. This increase in the head would add, at the most, 3.5 per cent to the peak power obtained from equation (1), bringing 826 MW up to 855 MW, closer to the value from the numerical model.

5.2 All channels open

For the true tidal regime, we consider three separate scenarios. In the first, turbines are simulated in the northwestern region of Johnstone Strait, leaving open the two main channels, Discovery Passage and Cordero Channel, that connect to the Strait of Georgia. This region of Johnstone Strait is located in series with the major branching of the flow into Discovery Passage and Cordero Channel and thus should give results similar to those for the one channel case. In the second scenario, we leave northwestern Johnstone Strait and Cordero Channel unmodified and simulate the presence of turbines in Discovery Passage, which has some of the largest tidal currents in the world [**14**] and is, therefore, a prime candidate for any future tidal current projects [**15**]. However, this channel not only has heavy

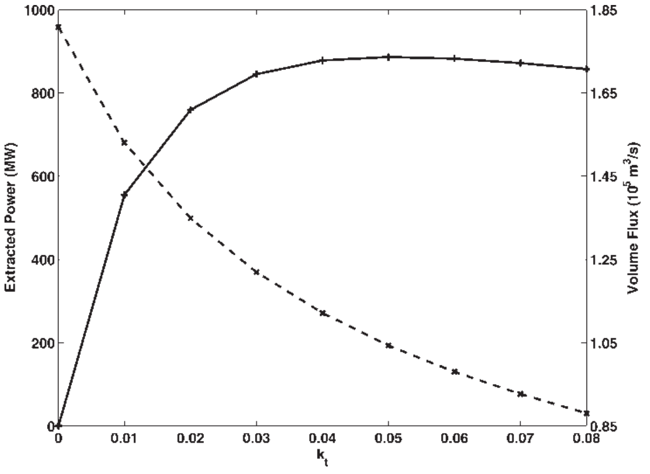


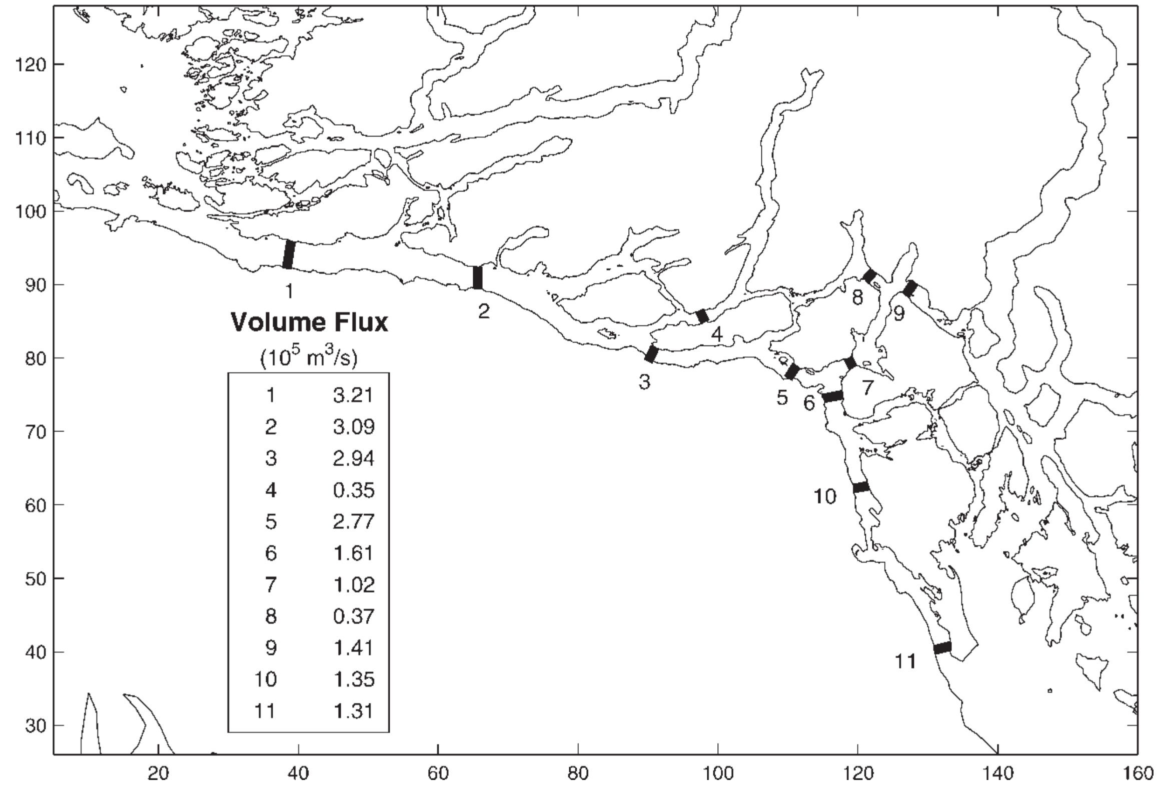
Fig. 5 Power dissipated by the addition of turbines in Discovery Passage, with Cordero Channel closed, as a function of increased friction coefficient. The solid line denotes the power dissipated (scale on left) and the dotted line denotes the change in volume flux through the channel (scale on right)

shipping traffic, but is also a major migration corridor for salmon returning to the Fraser River. Therefore, it would seem unrealistic to be able to extract the maximum possible tidal energy as part of the channel would need to be kept open. Hence, in a third scenario, we simulate turbines in Cordero Channel, which has relatively high currents in addition to being out of the way of major shipping traffic, while leaving northwestern Johnstone Strait and Discovery Passage unmodified. As shown in Fig. 6, Cordero Channel and Discovery Passage (i.e. transects 9 and 10, respectively) have comparable volume fluxes in the natural state, thus rendering previous single-channel theoretical assumptions invalid in our second and third scenarios.

For turbines simulated in northwestern Johnstone Strait (Fig. 3), our first scenario, the modelled peak power is 1335 MW (Fig. 7). This is in good agreement with equation (1), which predicts a maximum extractable power of 1320 MW using 3.11×10^5 m^3/s for $Q_{\max}$, 2.11 m for a, and $\gamma = 0.20$. In this section of Johnstone Strait, the volume flux varies between 3.00×10^5 and 3.21×10^5 m^3/s so 3.11×10^5 m^3/s is chosen as the mean maximum flux. The peak volume flux falls to 58 per cent of the value in the natural regime at maximum power extraction, again close to the expected 56 per cent. At maximum power, the height difference, a, increased to 2.18 m, which would increase the theoretical peak power, at the most, to 1363 MW.

Next, the bottom friction coefficient is slowly increased in Discovery Passage with Cordero Channel left unchanged, and the extracted power is shown in Fig. 8. The peak extractable power here is 401 MW. The same is then done for Cordero Channel with Discovery passage left unchanged, and the maximum extractable power is 277 MW as shown in Fig. 9. The peak volume flux drops to 57 and 58 per cent of the value in the natural state for Discovery Passage and Cordero Channel, respectively.

Using 2.11 m for a in equation (1), along with $\gamma = 0.20$ and 1.35×10^5 and 1.41×10^5 m^3/s for the peak volume fluxes in Discovery Passage and Cordero Channel, gives an estimated power potential of 573 and 598 MW, respectively. The single channel analytical model is inappropriate for these situations involving turbines in either Discovery Passage or Cordero Channel, however, due to the flow diverting into the channel without the turbines. This necessitates the need for a numerical model to accurately estimate the power potential, though it is possible that the single channel analytical model could be extended. In both of the cases evaluated above, at maximum power extraction the volume flux increases by 14 per cent in the channel without the



Volume Flux

(10^5 m^3/s)

1	3.21
2	3.09
3	2.94
4	0.35
5	2.77
6	1.61
7	1.02
8	0.37
9	1.41
10	1.35
11	1.31

Fig. 6 Volume flux through Johnstone Strait with no channels blocked. The axes give the horizontal distances in kilometres

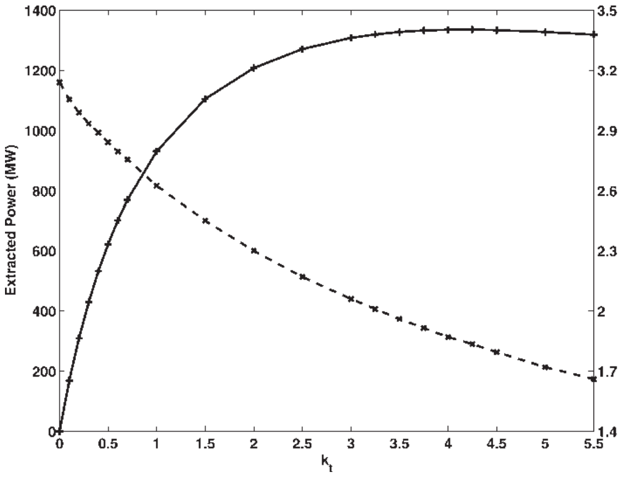


Fig. 7 Power extracted by the turbines in northwestern Johnstone Strait as a function of the extra friction coefficient. The solid line denotes the energy dissipation rate associated with the turbines (scale on left) and the dotted line denotes the change in peak volume flux through the channel (scale on right)

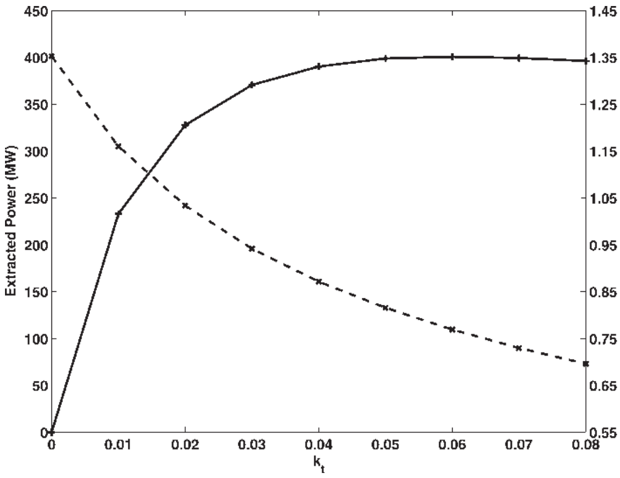


Fig. 8 Power extracted by the turbines in Discovery Passage as a function of the extra friction coefficient. The solid line denotes the energy dissipation rate associated with the turbines (scale on left) and the dotted line denotes the change in peak volume flux through the channel (scale on right)

turbines and the change in the height difference, a, is negligible.

5.3 Summary of results

The results of the turbine simulations along with the theoretical estimates for the maximum energy dissipation from equation (1) are shown in Table 1. For

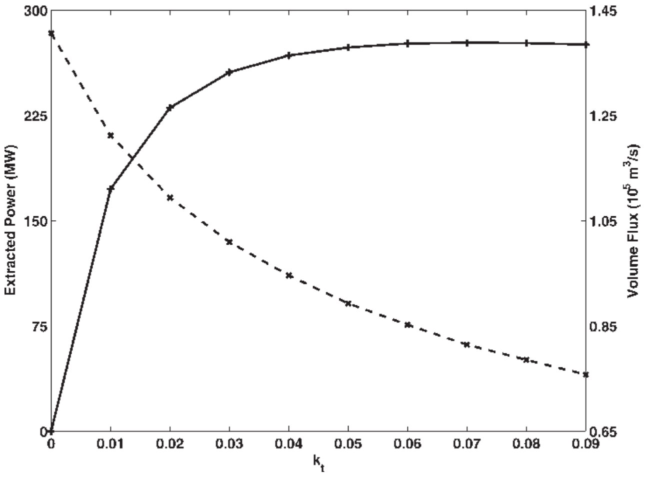


Fig. 9 Power extracted by the turbines in Cordero Channel as a function of the extra friction coefficient. The solid line denotes the energy dissipation rate associated with the turbines (scale on left) and the dotted line denotes the change in peak volume flux through the channel (scale on right)

Table 1 Results from simulating turbines in certain regions of Johnstone Strait

Location	Theoretical dissipation (MW)	Modelled dissipation (MW)	Percent vol. flux at max dissipation
Discovery Passage w/ Cordero closed	826	886	58
Johnstone Strait	1320	1335	58
Discovery Passage	573*	401	57
Cordero Channel	598*	277	58

Asterisks denote power estimates for channels where the water can be diverted and equation (1) is no longer valid.

the two cases with all the tidal flow going through the turbines there is good agreement between power estimates from equation (1) and numerical simulations. However, for the two cases where the flow can be diverted to a channel without turbines the theory is no longer valid and estimates cannot be made using equation (1). It is unclear why Cordero Channel, which has the greater volume flux in the natural regime, has a smaller power potential than Discovery Passage, even though both have roughly the same drop in volume flux, and the same increase in volume flux for the neighbouring channel, at peak power. Further study is required to discover the nature of this discrepancy.

5.4 Adding other constituents

These results for M2 can be extrapolated to account for the entire tide [**1**]. For a multi-frequency tide, e.g. $\zeta_0 = a\cos\omega t + a_1\cos\omega_1 t + a_2\cos\omega_2 t\cdots$, the extractable power is multiplied in the quasi-steady limit by $1 + \frac{9}{16}(r_1^2 + r_2^2\cdots)$, where $r_1 = a_1/a$, $r_2 = a_2/a$, … . and a is the M2 sinusoidal height difference between Alert Bay and Twin Islets. The factor 9/16 is replaced by 1 if the basic state is dominated by acceleration rather than friction [**1**], but we retain it, as the basic state here is closer to the quasi-steady, frictionally dominated limit. The scaling factor was computed using the harmonic constants for the eight major tidal constituents: M2, S2, N2, K2, K1, O1, P1, and Q1 were calculated through a harmonic analysis [**17**] of the observed time series [**11**] at these two locations. Accounting for these eight major constituents instead of just M2 will only increase the total power potential by a factor of 1.12.

6 FAR FIELD EFFECTS

To determine the far field effects, the tidal height amplitudes were compared between the natural regime and the regime at maximum power extraction for the three cases with the real geometry (i.e. excluding the first case where the side channels are blocked). The one channel case is not analysed here as it seems unlikely that Cordero Channel, Oksillo Channel, and Haskyn Channel would all be blocked off to force the flow through Discovery Passage.

We are assuming here that the prescribed tidal elevation at the open boundary, which generally lies beyond the edge of the continental shelf, is unchanged. This is likely to be a good assumption because any significant changes in deep water would lead to large transport changes which would be incompatible with the rest of the ocean. Further discussion of this issue is given by Garrett and Greenberg [**21**].

The variation in the phase lag of the tidal elevation is negligible outside Johnstone Strait. The maximum change is an increase in the phase lag of 10° in the area where the turbines are added. The currents also vary little outside Johnstone Strait with the maximum deviation being a decrease of 2 cm/s in Juan de Fuca Strait. In the Strait of Georgia, the currents slightly increase between 0 and 1 cm/s.

Extracting 1335 MW from Johnstone Strait has an appreciable impact on the far field tidal elevations. In the Strait of Georgia, there is a near uniform decrease in the M2 amplitude of 15 cm. In Juan de Fuca Strait, the M2 amplitude increases in the western end and decreases eastward. A lot of this change is due to the degenerate M2 amphidrome near Victoria [**22**] moving towards the Strait of Georgia.

Similar patterns arise when extracting 401 and 277 MW from Discovery Passage and Cordero Channel, respectively, but the magnitude of the changes is smaller. In fact, the magnitude appears to vary linearly with the amount of energy extracted, i.e. the far field effects, away from Johnstone Strait have the same shape as for extracting 1335 MW out of Johnstone Strait, but the tidal height amplitude variation is scaled down by 30 per cent (401/1335) and 20 per cent (277/1335) for turbines in Discovery Passage and Cordero Channel, respectively.

The effects of blocking Johnstone Strait completely are very similar to extracting 1335 MW out of Johnstone Strait with a slightly greater decrease in the M2 amplitude in the Strait of Georgia and Juan de Fuca Strait. At peak extraction, which has maximized the balance between the dissipative force and the flow through the turbines, the far field effects are similar to blocking off the channel completely so no water flows through.

Estimates of the far field effects using only M2 will be inaccurate if Johnstone Strait has an appreciable effect on the other constituents. The diurnal tidal amplitude is comparable to the semi-diurnal tide in most of the Strait of Georgia (the K1 amplitude is in fact larger than the M2 amplitude at Victoria). There may be significant changes in the diurnal tides as the resonant period is closer to the diurnal band than the semi-diurnal band [**23**]. As a result, estimating far field effects using only M2 is insufficient. Further work on this may be desirable.

7 DISCUSSION

We have shown that, when the flow cannot be diverted away from the channel with the turbines, the numerical results for the tidal power potential agree well with the analytic theory of Garrett and Cummins [**1**]. For these two scenarios, the estimates using equation (1) are both within 10 per cent of the numerical results. Small variations in the volume flux along the channel and the slight increase in the head with added friction will cause small discrepancies in the calculated power potential, but variations in the volume flux and head difference appear to be either negligible or cancel each other out. Thus the assumptions made by Garrett and Cummins [**1**] appear reasonable and their model useful.

If the flow can be diverted away from the turbines, the analytic theory [**1**] is no longer directly applicable. This is apparent for turbines added in Discovery Passage and Cordero Channel where the power potential is not predicted by equation (1) as it does not account for this diversion of the flow. These more complex channels can be addressed using a numerical model to estimate the maximum extractable power, though a semi-analytic extension of the basic theory could be undertaken. With flow diversion, equation (1) still gives an upper bound on the power potential, though this may not be very useful.

Given the support for the basic theory of Garrett and Cummins [**1**], it is clear that results rely on the model providing a good representation of the volume flux as well as the tidal elevation in the natural regime. Measurements of the M2 barotropic volume flux are available from current meter measurements in Johnstone Strait [**13**]. The peak volume flux was measured to be $2.6 \times 10^5\,\mathrm{m}^3/\mathrm{s}$ from five current meters along a transect at roughly the same location as transect 2 in Fig. 6. This, along with 2.12 m for a and 0.20 for γ, would result in equation (1) estimating the maximum power potential to be 1108 MW for the entire channel. This is less than from the model which has a larger volume flux of roughly $3.1 \times 10^5\,\mathrm{m}^3/\mathrm{s}$. Both observations and the model have uncertainties, so further work is needed to establish the correct volume flux. In particular, a more intensive current measurement program, using Acoustic Doppler Current Profilers instead of single current meters, could provide more accurate data on the tidal volume flux in the natural state. We emphasize that this would remove the sensitivity to the uncertainty of friction coefficients in the numerical model; as long as these are chosen so that the computed flux matches the observed value, they need not be accurate in every location.

In a study for BC HYDRO [**5**], the tidal current power potential was assessed for multiple sites along the BC coast, with the majority of these located in Johnstone Strait. The power potential at each site was estimated from the kinetic energy flux $(1/2)\rho u^3$ multiplied by the cross-sectional area. Only sites with current speeds greater than 2.4 m/s were chosen, excluding Seymour Narrows as the currents were deemed too high for present technology, and their estimated total power potential from 12 sites in Johnstone Strait was found to be 767 MW. This assessment looked at each site separately and assumed that extraction from one site will not affect extraction at another site, though this has been shown by Garrett and Cummins [**1**] and this study to be false. Also, in using the kinetic energy flux of the undisturbed flow as a metric for the maximum potential, changes to the flow due to the increased drag from energy extraction are neglected, quite apart from the fact that the kinetic energy flux varies from cross-section to cross-section.

Large current speeds are desirable for efficient turbine operation, and, for an isolated turbine, the extractable power is proportional to the kinetic energy flux of the basic flow through the area presented by the turbine. However, when turbines are present in the entire cross-section of a channel as a tidal fence, the maximum extractable energy is not proportional to the natural kinetic energy flux in any general way. The general analytic theory of Garrett and Cummins [**1**] has been shown here to be accurate in both predicting the change in the current from power extraction and determining the maximum power potential for a single channel where the water cannot be diverted away from the turbines. Specifically, the assumptions made in the analytical theory seem to be adequately valid when compared with the results from a numerical model.

Both the analytical model and the numerical model used here do use particular representations of the turbines. Future work will include investigating different ways to add friction to simulate turbines. One method would be to scale the turbine friction coefficient linearly with water depth so the body force will be constant for the whole turbine farm, i.e. $k_t = \alpha H$ where α is a constant and H is the water depth. This method has been applied to northwestern Johnstone Strait with nearly identical results to those obtained with the uniform friction coefficient increase.

Another important extension will be to increase the friction for only part of the channel in order to leave a portion open for shipping. In that case, of course, water can flow around the turbines rather than through them. Such an extension could be carried out using a two-dimensional model if the partial tidal fence occupies the whole water column in the vertical but is limited laterally, but a three-dimensional model is required if the turbines are confined near the sea floor. Preliminary investigations show that, in either of these scenarios, a turbine farm which is extensive in the along-channel direction is ineffective as the water will tend to avoid it and flow through the unrestricted part of the cross-section. Turbine fences would then need to be well spaced in the along-channel direction, allowing turbulent mixing to cause a recovery of the flow profile between fences. In this case, however, a loss of head is associated with the merging of flows that have, and have not, passed through a particular fence, giving less power than if the tidal fences occupy the entire channel [**24**].

Finally, we stress that the maximum extractable power predicted by Garrett and Cummins [**1**] and computed in this article would have to be reduced to allow for losses to drag on turbine support structures and to allow for the internal efficiency of the turbines themselves.

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